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A BULK INPUT QUEUEING SYSTEM WITH FEEDBACK AND SINGLE WORKING VACATION

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ABSTRACT

We study a bulk input Queueing system with feedback and single working vacation. The server provides service to customers, one by one, on a FCFS basis. Just after completion of his service a customer may leave the system or may opt to repeat his service, in which case this customer rejoins the head of the queue. Further, just after completion of a customer's service the server may take a single working vacation. Using supplementary variable technique we derive the probability generating function for the number of customers in the system and the average number of customers in the system. Some particular cases are discussed.

KEYWORDS: *Group Arrivals, Feedback, Single Working Vacation, Supplementary Variable Technique.*

AMS SUBJECT CLASSIFICATION NUMBER: *60K25, 60K30.*

I. INTRODUCTION

The queueing systems, which include the possibility of a customer after receiving one service, returning to the server for additional service, are called queues with feedback. Such a queueing models plays vital role in the area of computer networks, production systems, hospital management, super markets and banking etc. Formulation of queues with feedback mechanism was first introduced by Takacs (1963). Disney et.al. (1980, 1984), Simon (1984), Hunter (1985), Thangaraj and Vanitha (2010) are to mention few studied feedback queues.

In the literature of queueing systems with vacations has been discussed through a considerable amount of work in the recent past. Doshi (1990) has recorded prior work on vacation models and their applications in his survey paper. In recent years, few authors who were concentrated on vacation queues are Baba (1986), Madan and Gautam Choudhury (2005), Kalyanaraman and Pazhani Bala Murugan (2008) and Thangaraj and Vanitha (2010).

The feedback queueing systems with vacations has been studied by number of authors. To mention a few references we would name Takine.et.al.(1991), Boxma and Yechiali (1997), Kalyanaraman et.al. (2002), Lakshmi Srinivasan et. al. (2003), and Madan (2005).

Recently a class of semi-vacation policies has been introduced by Servi and Finn. Such a vacation is called working vacation(WV). The server works at a lower rate rather than completely stops service during a vacation. Servi and Finn (2002) studied an M/M/1 queue with multiple working vacation and obtained the probability generating function for the number of customers in the system and the waiting time distribution. Some other notable works were done by Wu and Takagi (2006). Tian et.al. (2008) Aftab Begum (2011) and Santhi and Pazhani Bala Murugan (2013).

In this paper, we study the bulk input Queueing system with feedback and single working vacation (SWV). The organization of the paper is as follows. In section 2, we described the model. In section 3, we obtained the steady state probability generating function. The performance measures are obtained in section 4 in section 5 some particular cases have been discussed and in section 6 conclusions are given.

II. NOTATIONS

λ	-	the mean arrival rate
η	-	the mean vacation rate
$S_b(x)$	-	the service time distribution
$s_b(x)$	-	the probability density function of $S_b(x)$
$S_b^*(\theta)$	-	the Laplace Stieltjes Transform
$S_v(x)$	-	the service time distribution during working vacation period
$s_v(x)$	-	the Probability density function of $S_v(x)$
$S_v^*(\theta)$	-	the Laplace Stieltjes Transform
p	-	the probability that a customer may repeat his service
$N(t)$	-	the system size at time t
$S_b^0(t)$	-	the remaining service time in not working vacation period
$S_v^0(t)$	-	the remaining service time in working vacation period
$Y(t)$	-	the server state
Q_0	-	the probability that the server is idle during the working vacation period
P_0	-	the probability that the server is idle during not working vacation period
$Q_n(x)$	-	the probability that there are n customers in the system and server is on working vacation and the remaining service time of the customer is x
$P_n(x)$	-	the probability that there are n customers in the system when the server is on not working vacation and the remaining service time of the customer is x
$Q_n^*(\theta)$	-	Laplace Stieltjes Transform of $Q_n(x)$
$P_n^*(\theta)$	-	Laplace Stieltjes Transform of $P_n(x)$
$P(z)$	-	the probability generating function for the number of customers in the system irrespective of the server state
$P_b(z)$	-	the probability generating function for the number of customers in the system when the server is on not working vacation period
$P_v(z)$	-	the probability generating function for the number of customers in the system when the server is on working vacation period
L_b	-	the mean number of customers in the system when the server is on not working vacation period
L_v	-	the mean number of customers in the system when the server is on working vacation period

III. THE MODEL DESCRIPTION

We assume the following to describe the queueing model of our study.

- Customers arrive in batches according to a Compound Poisson process with mean arrival rate $\lambda(>0)$.
- The batch size X is a random variable and $\Pr(X = n) = g_n$, n = 1, 2, 3, … with probability generating function $X(z) = \sum_{k=1}^{\infty} g_k z^k$ and the first and second factorial moments of X are defined by $g^{(1)} = E(X) = X'(1)$ and $g^{(2)} = E[X(X-1)] = X''(1)$ respectively.
- The service discipline is FCFS.

- The service time is general distribution. Let $S_b(x)$, $s_b(x)$ and $S_b^*(\theta)$ be the probability distribution function, the probability density function and the Laplace-Stieltjes transform (LST) of the service time S_b.
- Whenever the system becomes empty at a service completion instant the server starts working vacation and the duration of the vacation time follows an exponential distribution with rate η. At a vacation completion instant, if there are customers in the system the server will start a new busy period. Otherwise he waits until a customer arrive. This type of vacation policy is called single working vacation. During the working vacation period the server provides the service with the service time S_v of a typical customer follows a general distribution with the distribution function $S_v(x)$[$s_v(x)$ the probability density function and $S_v^*(\theta)$, the LST].
- After completion of his service a customer may like to repeat his service with probability p or may leave the system with probability $q = 1-p$ in both not working vacation period and working vacation period.
- Further, it is noted that the service interrupted at the end of a vacation is lost and it is restarted with a different distribution at the beginning of the following service period. Inter arrival times, service times and working vacation times are mutually independent of each other.

The system size distribution at an arbitrary time will be treated by the supplementary variable technique. That is, from the joint distribution of the queue length and the remaining service time of the customer in service if the server is busy, or the remaining service time of the customer if the server is on Working Vacation. Assuming that the system is in steady state condition. Let us define the following random variables.

$N(t)$ - the system size at time t.

$S_b^0(t)$ - the remaining service time in not WV period.

$S_v^0(t)$ - the remaining service time in WV period.

$$Y(t) - \begin{cases} 0 & \text{if the server is idle on vacation} \\ 1 & \text{if the server is idle in not WV period} \\ 2 & \text{if the server is busy in not WV period} \\ 3 & \text{if the server is busy in WV period} \end{cases}$$

so that the supplementary variables $S_b^0(t)$ and $S_v^0(t)$ are introduced in order to obtain bivariate Markov Process $\{N(t), \partial(t); t \geq 0\}$, where

$$\partial(t) = \begin{cases} S_b^0(t) & \text{if } Y(t)=2 \\ S_v^0(t) & \text{if } Y(t)=3 \end{cases}$$

We define the following limiting probabilities:

$$Q_0 = \lim_{t\to\infty} \Pr\{N(t) = 0, Y(t) = 0\}$$

$$P_0 = \lim_{t\to\infty} \Pr\{N(t) = 0, Y(t) = 1\}$$

$$Q_n(x) = \lim_{t\to\infty} \Pr\{N(t) = n, Y(t) = 3, x < S_v^0(t) \leq x + dx\}, n \geq 1.$$

$$P_n(x) = \lim_{t\to\infty} \Pr\{N(t) = n, Y(t) = 2, x < S_b^0(t) \leq x + dx\}, n \geq 1.$$

We define the Laplace Stieltjes Transform and the probability generating functions as follows:

$$S_b^*(\theta) = \int_0^\infty e^{-\theta x} s_b(x)dx \qquad S_v^*(\theta) = \int_0^\infty e^{-\theta x} s_v(x)dx$$

$$Q_n^*(\theta) = \int_0^\infty e^{-\theta x} Q_n(x)dx \qquad P_n^*(\theta) = \int_0^\infty e^{-\theta x} P_n(x)dx$$

$$Q^*(z,\theta) = \sum_{n=1}^{\infty} z^n Q_n^*(\theta) \qquad Q(z,0) = \sum_{n=1}^{\infty} z^n Q_n(0)$$

$$P^*(z,\theta) = \sum_{n=1}^{\infty} z^n P_n^*(\theta) \qquad P(z,0) = \sum_{n=1}^{\infty} z^n P_n(0)$$

IV. THE SYSTEM SIZE DISTRIBUTION

By considering the steady state, we have the following system of the differential difference equations.

$$(\lambda + \eta)Q_0 = qP_1(0) + qQ_1(0);\ n = 0 \tag{1}$$

$$-\frac{d}{dx} Q_1(x) = -(\lambda + \eta)Q_1(x) + qQ_2(0)s_v(x) + pQ_1(0)s_v(x) + \lambda g_1 Q_0 s_v(x)\ ;\ n=1 \tag{2}$$

$$-\frac{d}{dx} Q_n(x) = -(\lambda + \eta)Q_n(x) + qQ_{n+1}(0)s_v(x) + pQ_n(0)s_v(x) + \lambda g_n Q_0 s_v(x) + \lambda \sum_{k=1}^{n-1} g_k Q_{n-k}(x)\ ;\ n \geq 2 \tag{3}$$

$$\lambda P_0 = \eta Q_0 \tag{4}$$

$$-\frac{d}{dx} P_1(x) = -\lambda P_1(x) + qP_2(0)s_b(x) + pP_1(0)s_b(x) + \lambda g_1 P_0 s_b(x) + \eta s_b(x) \int_0^{\infty} Q_1(y)dy\ ;\ n = 1 \tag{5}$$

$$-\frac{d}{dx} P_n(x) = -\lambda P_n(x) + qP_{n+1}(0)s_b(x) + pP_n(0)s_b(x) + \lambda g_n P_0 s_b(x) + \lambda \sum_{k=1}^{n-1} g_k P_{n-k}(x) + \eta s_b(x) \int_0^{\infty} Q_n(y)dy\ ;\ n > 1 \tag{6}$$

Taking the LST of (2), (3), (5) and (6) we have

$$\theta Q_1^*(\theta) - Q_1(0) = (\lambda+\eta) Q_1^*(\theta) - qQ_2(0) S_v^*(\theta) - pQ_1(0) S_v^*(\theta) - \lambda g_1 Q_0 S_v^*(\theta)\ ;\ n = 1 \tag{7}$$

$$\theta Q_n^*(\theta) - Q_n(0) = (\lambda+\eta) Q_n^*(\theta) - qQ_{n+1}(0) S_v^*(\theta) - pQ_n(0) S_v^*(\theta) - \lambda g_n Q_0 S_v^*(\theta) - \lambda \sum_{k=1}^{n-1} g_k Q_{n-k}^*(\theta);\ n>1 \tag{8}$$

$$\theta P_1^*(\theta) - P_1(0) = \lambda P_1^*(\theta) - qP_2(0) S_b^*(\theta) - pP_1(0) S_b^*(\theta) - \lambda g_1 P_0 S_b^*(\theta) - \eta S_b^*(\theta) \int_0^{\infty} Q_1(y)\,dy;\ n=1 \tag{9}$$

$$\theta P_n^*(\theta) - P_n(0) = \lambda P_n^*(\theta) - qP_{n+1}(0) S_b^*(\theta) - pP_n(0) S_b^*(\theta) - \lambda g_n P_0 S_b^*(\theta) - \lambda \sum_{k=1}^{n-1} g_k P_{n-k}^*(\theta) - \eta S_b^*(\theta) \int_0^{\infty} Q_n(y)\,dy;\ n>1 \tag{10}$$

z^n times (8) summing over n from 2 to ∞ is added up with z times (7) we get

$$[\theta - (\lambda - \lambda X(z) + \eta)]Q^*(z,\theta) = Q(z, 0) \left[\frac{z - S_v^*(\theta) + p(1-z)S_v^*(\theta)}{z}\right] - S_v^*(\theta)\,[\lambda X(z)Q_0 - qQ_1(0)] \tag{11}$$

Inserting $\theta = (\lambda - \lambda X(z) + \eta)$ in (11) we get

$$Q(z, 0) = \frac{zS_v^*(\lambda-\lambda X(z)+\eta)[\lambda Q_0 X(z) - qQ_1(0)]}{z - S_v^*(\lambda-\lambda X(z)+\eta) + p(1-z)S_v^*(\lambda-\lambda X(z)+\eta)} \quad (12)$$

The denominator of the above equation has a unique root z_1 in (0, 1). Therefore
$qQ_1(0) = \lambda X(z_1)Q_0$
Substituting this in (12) we have

$$Q(z, 0) = \left[\frac{\lambda z Q_0 S_v^*(\lambda-\lambda X(z)+\eta)(X(z)-X(z_1))}{z - S_v^*(\lambda-\lambda X(z)+\eta) + p(1-z)S_v^*(\lambda-\lambda X(z)+\eta)}\right] \quad (13)$$

Substituting (13) in (11) and inserting θ = 0, we get

$$Q^*(z,0) = \frac{\lambda z Q_0 (X(z)-X(z_1))[1-S_v^*(\lambda-\lambda X(z)+\eta)]}{(\lambda-\lambda X(z)+\eta)[z - S_v^*(\lambda-\lambda X(z)+\eta) + p(1-z)S_v^*(\lambda-\lambda X(z)+\eta)]} \quad (14)$$

z^n times (10) summing over n from 2 to ∞ is added up with z times (9), we get

$$[\theta - (\lambda-\lambda X(z))]P^*(z,\theta) = P(z, 0)\left[\frac{z - qS_b^*(\theta) - pzS_b^*(\theta)}{z}\right] - S_b^*(\theta)$$

$$[\lambda P_0 X(z) + \eta Q^*(z,0) - qP_1(0)] \quad (15)$$

Inserting $\theta = (\lambda - \lambda X(z))$ in (15), we get

$$P(z, 0) = \frac{zS_b^*(\lambda-\lambda X(z))[\eta Q^*(z,0) + \lambda X(z)P_0 - qP_1(0)]}{z - S_b^*(\lambda-\lambda X(z)) + p(1-z)S_b^*(\lambda-\lambda X(z))} \quad (16)$$

Substituting (16) and $qP_1(0) = [\eta + \lambda(1-X(z_1))]Q_0$ in (15) and inserting θ = 0, we get

$$\begin{aligned} P^*(z, 0) = \frac{Q_0}{D_1(z)D_2(z)} \Big\{ & z(1-S_b^*(\lambda-\lambda X(z)))\{\eta\lambda z(X(z)-X(z_1)) \\ & x(1-S_v^*(\lambda-\lambda X(z)+\eta)) - (\eta(1-X(z)) + \lambda(1-X(z_1))) \\ & x(\lambda-\lambda X(z)+\eta)x[z-(1-p)S_v^*(\lambda-\lambda X(z)+\eta) \\ & - pzS_v^*(\lambda-\lambda X(z)+\eta)]\}\Big\} \end{aligned} \quad (17)$$

where

$$D_1(z) = (\lambda - \lambda X(z))[z - S_v^*(\lambda-\lambda X(z)+\eta) + p(1-z)\, S_v^*(\lambda-\lambda X(z)+\eta)] \quad (18)$$

$$D_2(z) = (\lambda - \lambda X(z)+\eta)\, [z - S_b^*(\lambda-\lambda X(z)) + p(1-z)\, S_b^*(\lambda-\lambda X(z))] \quad (19)$$

We define

$P_B(z) = P^*(z, 0) + P_0$

as the probability generating function for the number of customers in the system when the server is on not WV period and

$P_V(z) = Q^*(z, 0) + Q_0$

as the probability generating function for the number of customers in the system when the server is on WV period then

$$P(z) = P_B(z) + P_V(z) \quad (20)$$

as the probability generating function for the number of customers in the system. We shall now use the normalizing condition P(1) = 1 to determine the only unknown Q_0, which appears in (20). Substituting z = 1 in (20) and using L'hospital's rule we obtain

$$Q_0 = \frac{1}{\left[\frac{(\lambda-\lambda X(z_1)+\eta)}{\eta} + \frac{\eta}{\lambda} - \frac{\rho_b(1-X(z_1))S_v^*(\eta)}{E(X)(1-S_v^*(\eta))}\right]} - \frac{\rho_b}{(1-p)\left[\frac{(\lambda-\lambda X(z_1)+\eta)}{\eta} + \frac{\eta}{\lambda} - \frac{\rho_b(1-X(z_1))S_v^*(\eta)}{E(X)(1-S_v^*(\eta))}\right]} \quad (21)$$

where $\rho_b = \lambda E(X)E(S_b)$, $E(S_b)$ is the mean service time. From (21) we obtain the system stability condition,

$$\rho_b < 1. \qquad (22)$$

V. PERFORMANCE MEASURES

Mean System Length

Let L_v and L_b denote the mean system size during the working vacation and not working vacation period respectively. Then

$$Lv = \frac{d}{dz} P_v(z) \text{ at } z = 1$$

$$= \frac{d}{dz}\left\{\frac{N_1(z)}{D(z)}\lambda Q_0\right\} \text{ at } z = 1$$

where

$$N_1(z) = z(X(z) - X(z_1))\,[1 - S_v^*(\lambda - \lambda X(z) + \eta)]$$

$$D(z) = (\lambda - \lambda X(z) + \eta)[z - (1-p)\, S_v^*(\lambda - \lambda X(z) + \eta) - pz\, S_v^*(\lambda - \lambda X(z) + \eta)]$$

Therefore $$L_v = \frac{\lambda Q_0 \{D(1)N_1'(1) - D'(1)N_1(1)\}}{(D(1))^2}$$

where $N_1(1) = (1 - X(z_1))(1 - S_v^*(\eta))$

$$N_1'(1) = (1 - S_v^*(\eta))\, E(X) + (1 - X(z_1))[1 - S_v^*(\eta) + \lambda E(X)\, S_v^{*'}(\eta)]$$

$$D(1) = \eta[1 - S_v^*(\eta)]$$

$$D'(1) = -\lambda E(X)[1 - S_v^*(\eta)] + \eta[1 + \lambda E(X)\, S_v^{*'}(\eta) - p S_v^*(\eta)]$$

$$L_b = \frac{d}{dz} P_B(z) \text{ at } z = 1$$

$$= \frac{d}{dz}\left\{\frac{(N_2(z)N_3(z))}{D_1(z)D_2(z)}\lambda Q_0\right\} \text{ at } z = 1$$

where

$$N_2(z) = z(1 - S_b^*(\lambda - \lambda X(z)))$$

$$N_3(z) = \{\, \eta\lambda z(X(z) - X(z_1))(1 - S_v^*(\lambda - \lambda X(z) + \eta)) - [\lambda(1 - X(z_1)) + \eta(1 - X(z))](\lambda - \lambda X(z) + \eta) [(z - S_v^*(\lambda - \lambda X(z) + \eta)) + p(1-z)S_v^*(\lambda - \lambda X(z) + \eta)]\}$$

$D_1(z)$ and $D_2(z)$ are respectively given in equations (18) and (19).

Therefore $$L_b = \frac{\lambda Q_0}{2[D_1'(1)]^2[D_2'(1)]^2}\{D_1'(1)D_2'(1)[N_2''(1)N_3'(1) + N_3''(1)N_2'(1)] - N_2'(1)N_3'(1)[D_1''(1)D_2'(1) + D_1'(1)D_2''(1)]\}$$

Where $N_2^1(1) = -\lambda E(X)E(S_b)$

$$N_2''(1) = -2\lambda E(X)E(S_b) - (\lambda E(X))^2 E(S_b^2) - \lambda E(S_b)E([X(X-1)]$$

$$N_3'(1) = \lambda E(X)(1 - S_v^*(\eta))\,(\eta + \lambda(1 - X(z_1))) - (1-p)\eta\lambda\,(1 - X(z_1))S_v^*(\eta) + \eta^2 E(X)\,(1 - S_v^*(\eta))$$

$$N_3''(1) = 2\{\eta\lambda E(X)(1 - S_v^*(\eta))(1 - E(X)) + (1-p)\eta\lambda^2 E(X)(1 - X(z_1))S_v^{*'}(\eta) + \eta(\lambda E(X))^2 S_v^{*'}(\eta) + E(X)(\lambda^2(1 - X(z_1)) + \eta^2)[1 + \lambda E(X)S_v^{*'}(\eta) - pS_v^*(\eta)]\}$$

$$+E(X(X-1))(1-S_v^*(\eta))[\eta\lambda+\eta^2+\lambda^2(1-X(z_1))]$$

$$D_1'(1)=-\lambda E(X)[1-S_v^*(\eta)]$$

$$D_1''(1)=-2\lambda E(X)[1+\lambda E(X)S_v^{*'}(\eta)-pS_v^*(\eta)]-\lambda E(X(X-1))(1-S_v^*(\eta))$$

$$D_2'(1)=\eta[1-p-\lambda E(X)E(S_b)]$$

$$D_2''(1)=-2\lambda E(X)[1-p-\lambda E(X)E(S_b)]-\eta[(\lambda E(X))^2E(S_b^2)+2p\lambda E(X)E(S_b)$$
$$+\lambda E(S_b)E(X(X-1))]$$

where $E(S_b)$ is the mean service time. $E(S_b^2)$ is the second moment of the service time.

VI. PARTICULAR CASES

Case i: If no customer receives the feedback service then on setting p = 0 we get P(z) as

$P(z) = P_B(z) + P_V(z)$ (23)

where

$$P_B(z)=\frac{\begin{bmatrix}Q_0\left\{\lambda z(1-S_b^*(\lambda-\lambda X(z)))[\eta\lambda z(X(z)-X(z_1))(1-S_v^*(\lambda-\lambda X(z)+\eta))\right.\\-(\lambda(1-X(z_1))+\eta(1-X(z)))(\lambda-\lambda X(z)+\eta)(z-S_v^*(\lambda-\lambda X(z)+\eta))]\\\left.+\eta(\lambda-\lambda X(z))(\lambda-\lambda X(z)+\eta)(z-S_b^*(\lambda-\lambda X(z)))(z-S_v^*(\lambda-\lambda X(z)+\eta))\right\}\end{bmatrix}}{\lambda(\lambda-\lambda X(z))(\lambda-\lambda X(z)+\eta)(z-S_v^*(\lambda-\lambda X(z)+\eta))(z-S_b^*(\lambda-\lambda X(z)))}$$

$$P_V(z)=\frac{\begin{bmatrix}Q_0\left\{\lambda z(X(z)-X(z_1))(1-S_v^*(\lambda-\lambda X(z)+\eta))\right.\\\left.+(\lambda-\lambda X(z)+\eta)(z-S_v^*(\lambda-\lambda X(z)+\eta))\right\}\end{bmatrix}}{(\lambda-\lambda X(z)+\eta)(z-S_v^*(\lambda-\lambda X(z)+\eta))}$$

$$Q_0=\frac{1-\rho_b}{\left[\frac{(\lambda-\lambda X(z_1)+\eta)}{\eta}+\frac{\eta}{\lambda}-\frac{\rho_b S_v^*(\eta)(1-X(z_1))}{E(X)(1-S_v^*(\eta))}\right]}$$

where $\rho_b = \lambda E(X)E(S_b)$.

Equation (23) is well known generating function of the steady state queue length distribution of an $M^X/G/1$ queue with single working vacation.

Case ii: If the server never takes a vacation then taking limit as $\eta\to\infty$ we get

P(z) as $P(z) = P_B(z) + Q_0$ (24)

$$P_B(z)=\frac{z(S_b^*(\lambda-\lambda X(z))-1)}{z-S_b^*(\lambda-\lambda X(z))+p(1-z)S_b^*(\lambda-\lambda X(z))}Q_0$$

where $Q_0 = 1 - \rho_b$.

Equation (24) is well known generating function of the steady state queue length distribution of an $M^X/G/1$ feedback queue.

VII. CONCLUSIONS

The analysis carried out in bulk input queueing system with feedback and single working vacation is to obtain the probability generating function for the number of customers in the system.

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