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ANALYSIS OF A NON MARKOVIAN QUEUE WITH RESTRICTED ADMISSIBILITY AND OPTIONAL TYPES OF REPAIR

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ABSTRACT

This paper investigates the steady state behaviour of a batch arrival queueing model of two types of service with restricted admissibility policy, Bernoulli feedback service and random breakdown. Here the vacation is Bernoulli scheduled. On completion of a service, the server may go for a vacation with probability ω or continue to stay in the system with probability 1-ω. Breakdown may occur at any instant during the service. Optional Repair process follows immediately. Steady state results are obtained explicitly. Also the performance measures are derived.

KEY WORDS: *Types of service, restricted admissibility, Bernoulli schedule vacation, breakdown and optional repair policy.*

I. INTRODUCTION

In Queueing theory, several contributions have been made by many authors in the research study of batch arrival queueing models with breakdowns and repairs. Queue systems with batch arrival have been analyzed by many researchers including Levy and Yechiali[1], Keilson and Servi[2].
V.Thangaraj and S. Vanitha[3] studied a single server queue with two types of services. Moreover K.C.Madan[4] made an analysis on a single server queueing model with two stages of service.In that paper, he discussed the concept of binomial schedule server vacation. B.Krishnakumar, A. Vijayakumar and D. Arividainambi,[5] explained the concept of retrial in a queue with two phases of service.Choudhary and Madan[6].Survey on vacation queues was done by Doshi [8]. Choi et al.[11] analyzed two phase queueing system with vacation and Bernoulli feedback. Restricted admissibility policy is proposed on batch arrival queueing model by Madan and Abu dayyeh[10]. Badamchi Zadeh [7] investigated a batch arrival queuing system with two phases of service, second service is optional and restricted admissibility..K.C Madan and G.Chodhury[9] studied a queue on restricted admissibility. K.C Madan, [12] analysed a non markovian queue with optional service.V.Thangaraj and S. Vanitha [13] made a survey on a non markovian queue with Two stage heterogeneous service, compulsory server vacation and random breakdowns. Senthilkumar and Arumuganathan[14] studied a retrial queue with two phase service and service interruption. Arivudainambi and P. Godhandaraman[15],introduced a new concept of K optional vacation in his Retrial Queueing model with Two Phases of Service.
In this paper we study a batch arrival queuing model with two types of service and restricted admissibility. After the completion of service the server follows a Bernoulli schedule vacation. If the customer is dissatisfied with the service, the customer may opt for a feedback service. Once the system gets breakdown, it enters a repair process. Based on the repair, repair process is followed. Steady state solution and performance measures are derived explicitly.
The rest of the paper is organized as follows: Mathematical description of the model is given in section 2, Differential equations for the model is framed in section3, time dependant solution is

obtained in section 4, Steady state results are obtained in section 5, Performance measures for the model is described in section 6 and the model is concluded in section7.

II. MATHEMATICAL DESCRIPTION OF THE MODEL

a) Customers arrive at the system in batches of variable size in a compound Poisson process and they are provided one by one service on a 'first come'-first served basis. Let $\lambda c_i (i = 1,2,3 \ldots.)$ be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t + dt)$, where $0 \leq c_i \leq 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches. All the batches are not allowed to join the system at all times. Let r_1 be the probability that an arriving batch will be allowed to join the system during server's non vacation period. Let r_2 be the probability that an arriving is allowed during vacation period.

b)Two types of service is provided. The service time follows general(arbitrary)distribution with distribution function $B_i(s)$ and density function $b_i(s)$. Let $\mu_i(x)dx$ be the conditional probability density of service completion during the interval $(x, x + dx)$,given that the elapsed time is x,so that

$$\mu_i(x) = \frac{b_i(x)}{1 - B_i(x)} \qquad i = 1 \; or \; 2 \tag{1}$$

and therefore

$$b_i(s) = \mu_i(s) e^{-\int_0^s \mu_i(x)dx} \quad i = 1 \; or 2 \tag{2}$$

c) As soon as a service is completed, the server may go for a vacation with probability ω or it may continue to serve the next customer $(1 - \omega)$.

d)The server's vacation time follows a general(arbitrary) distribution with distribution function $V(s)$ and density function $v(s)$. Let $\vartheta(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x + dx)$,so that

$$\vartheta(x) = \frac{v(x)}{1 - V(x)} \tag{3}$$

And, therefore

$$v(s) = \vartheta(s) e^{-\int_0^s \vartheta(x)dx} \tag{4}$$

e) The system may breakdown at random and breakdowns are assumed to occur according to a Poisson stream with mean breakdown rate $\alpha > 0$

f)The repair time follows exponential distribution with mean repair rate $\beta > 0$.

III. DIFFERENTIAL DIFFERENCE EQUATIONS

The model is then, governed by the following set of differential-difference equations:.

$$\frac{\partial}{\partial x} P_n^{(1)}(x,t) + \frac{\partial}{\partial t} P_n^{(1)}(x,t) + (\lambda + \mu_1(x) + \alpha) P_n^{(1)}(x,t) = \lambda(1 - r_1) P_n^{(1)}(x,t)$$
$$+\lambda r \textstyle\sum_{k=1}^{n} c_k P_{n-k}^{(1)}(x,t), n \geq 1 \tag{5}$$

$$\frac{\partial}{\partial x} P_n^{(2)}(x,t) + \frac{\partial}{\partial t} P_n^{(2)}(x,t) + (\lambda + \mu_1(x) + \alpha) P_n^{(2)}(x,t) = \lambda(1 - r_1) P_n^{(2)}(x,t)$$
$$+\lambda r \textstyle\sum_{k=1}^{n} c_k P_{n-k}^{(2)}(x,t), n \geq 1 \tag{5a}$$

$$\frac{\partial}{\partial x} P_0^{(1)}(x,t) + \frac{\partial}{\partial t} P_0^{(1)}(x,t) + (\lambda + \mu(x) + \alpha) P_0^{(1)}(x,t) = \lambda(1 - r_1) P_0^{(1)}(x,t) \tag{6}$$

$$\frac{\partial}{\partial x} P_0^{(2)}(x,t) + \frac{\partial}{\partial t} P_0^{(2)}(x,t) + (\lambda + \mu(x) + \alpha) P_0^{(2)}(x,t) = \lambda(1 - r_1) P_0^{(2)}(x,t) \tag{6a}$$

$$\frac{\partial}{\partial x} V_n(x,t) + \frac{\partial}{\partial t} V_n(x,t) + \big(\lambda + v(x)\big) V_n(x,t) = \lambda(1 - r_2) V_n(x,t)$$

$$+\lambda r_2 \sum_{i=1}^{n-1} c_i V_{n-i}^{(i)}(x,t)\; n \geq 1 \qquad (7)$$

$$\frac{\partial}{\partial x} V_0(x,t) + \frac{\partial}{\partial t} V_0(x,t) + \big(\lambda + \gamma(x)\big) V_0(x,t) = \lambda(1-r_2) V_0(x,t) \qquad (8)$$

$$\frac{\partial}{\partial x} R_n^{(i)}(x,t) = -(\lambda+\beta)\frac{\partial}{\partial t} R_n^{(i)}(x,t) + \lambda(1-r_1)R_n^{(i)}(x,t)$$
$$+\lambda r_1 \sum_{i=1}^{n-1} c_i R_{n-i}^{(i)}(x,t) + \alpha \int_0^\infty P_{n-1}^{(i)}(x,t) + \alpha \int_0^\infty P_{n-1}^{(i)}(x,t) \qquad (9)$$

$$\frac{\partial}{\partial x} R_0^{(i)}(x,t) = -(\lambda+\beta)\frac{\partial}{\partial t} R_0^{(i)}(x,t) + \lambda(1-r_1)R_0^{(i)}(x,t) \qquad (10)$$

$$\frac{d}{dt} Q(t) = -\lambda r_1 Q(t)(1-\alpha) + \lambda r_1(1-r_2)Q(t) + r_2\beta R_0^{(i)}$$
$$+r_1 \int_0^\infty V_0(x,t)\gamma(x)dx\, r_2(1-\omega) q \int_0^\infty P_0(x,t)\mu(x)dx \qquad (11)$$

The above equations are to be solved subject to the following boundary conditions:

$$r_2 P_n^{(1)}(0,t) = r_2(1-\omega)\left\{p\int_0^\infty P_n^{(1)}(x,t)\mu_1(x)dx + q\int_0^\infty P_{n+1}^{(1)}(x,t)\mu_1(x)dx\right\}$$
$$+r_1 \int_0^\infty V_{n+1}(x,t)v(x)dx + r_2\beta R_{n+1}^{(i)}(t) + \lambda r_1 r_2 C_{n+1} Q(t), n \geq 0 \qquad (12)$$

$$P_n^{(2)}(0,t) = \int_0^\infty P_n^{(1)}(x,t)\mu_1(x)dx\,,\; n \geq 0 \qquad (12a)$$

$$r_1 V_n(0,t) = r_2 \omega \int_0^\infty P_n^{(2)}(x,t)\mu_2(x)dx, \quad n \geq 0 \qquad (13)$$

Assuming there are no customers in the system initially so that the server is idle

$$V_0(0) = 0;\; V_n(0) = 0; Q(0) = 1;\; P_n(0) = 0, n = 0,1,2 \ldots \qquad (14)$$

IV. THE TIME-DEPENDENT SOLUTION

In this section we obtain the transient solution for the above set of differential-difference equations. We define the probability generating functions,

$$P_q(x,z,t) = \sum_{n=0}^\infty z^n P_n(x,t); P_q(z,t) = \sum_{n=0}^\infty z^n P_n(t)$$
$$V_q(x,z,t) = \sum_{n=0}^\infty z^n V_n(x,t); V_q(z,t) = \sum_{n=0}^\infty z^n V_n(t)$$
$$R_q^{(i)}(z,t) = \sum_{n=0}^\infty z^n R_n^{(i)}(t) C(z) = \sum_{n=1}^\infty c_n z^n \qquad (15)$$

Which are convergent inside the circle by $z \leq 1$ and define the Laplace transform of a function f(t) as

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt \qquad (16)$$

Taking the Laplace transform of equations (1) to (13) and using (15) we obtain

$$\frac{\partial}{\partial x}\bar{P}_n^{(1)}(x,s) + (s+\lambda+\mu_1(x)+\alpha)\bar{P}_n^{(1)}(x,s) = \lambda(1-r_1)\bar{P}_n^{(1)}(x,s) + \lambda r_1 \sum_{k=1}^{n} c_k \bar{P}_{n-k}^{(1)}(x,s),$$
$$n \geq 1 \qquad (17)$$

$$\frac{\partial}{\partial x}\bar{P}_0^{(1)}(x,s) + (s+\lambda+\mu_1(x)+\alpha)\bar{P}_0^{(1)}(x,s) = \lambda(1-r_1)\bar{P}_0^{(1)}(x,s) \qquad (18)$$

$$\frac{\partial}{\partial x}\bar{V}_n(x,s)+(s+\lambda+\mu_1(x)+\alpha)\bar{V}_n(x,s)=\lambda(1-r_2)\bar{V}_n(x,s)+\lambda r_2\sum_{i=1}^{n-1}c_i(x,s)\bar{V}_{n-i} \quad (19)$$

$$\frac{\partial}{\partial x}\bar{V}_0(x,s)+\big(s+\lambda+v(x)\big)\bar{V}_0(x,s)=\lambda(1-r_2)\bar{V}_0(x,s) \quad (20)$$

$$(s+\lambda+\beta)\bar{R}_n^{(i)}(x,s)=\lambda(1-r_1)\bar{R}_n^{(i)}(x,s)+\lambda r_1\sum_{i=1}^{n-1}c_i\,\bar{R}_{n-i}^{(i)}(x,s)+\alpha\int_0^\infty \bar{P}_{n-1}^{(1)}(x,s) + \int_0^\infty \bar{P}_{n-1}^{(1)}(x,s) \quad (21)$$

$$(s+\lambda+\beta)\bar{R}_0^{(i)}(x,s)=\lambda(1-r_1)\bar{R}_0^{(i)}(x,s) \quad (22)$$

$$[s+\lambda r_1]\bar{Q}(s)=1+\lambda r_1(1-r_2)Q(s)+\beta r_2\bar{R}_0^{(i)}(s) +r_2(1-\omega)q\int_0^\infty \bar{P}_0^{(2)}(x,s)\mu(x)dx+r_1\int_0^\infty \bar{V}_0(x,s)v(x)dx \quad (23)$$

For boundary conditions,

$$r_2\bar{P}_n^{(1)}(0,s)=(1-\omega)r_2\left\{p\int_0^\infty \bar{P}_n^{(2)}(x,s)\mu_2(x)dx+q\int_0^\infty \bar{P}_{n+1}^{(2)}(x,s)\mu_2(x)dx\right\} +r_1\int_0^\infty \bar{V}_{n+1}(x,s)dx+\int_0^\infty \Upsilon_3(x)\bar{V}_{n+1}^{(3)}(x,s)dx+p(1-r)\int_0^\infty \mu_N(x)\bar{P}_n^{(N)}(x,s)dx + (1-p)(1-r)\int_0^\infty \bar{P}_{n+1}^{(N)}(x,s)\mu_N(x)dx,\ n\ge 0 \quad (24)$$

$$\bar{P}_n^{(2)}(0,s)=\int_0^\infty \mu_1(x)\bar{P}_n^{(1)}(x,s)dx, n\ge 0 \quad (24a)$$

$$r_1\bar{V}_n(0,s)=r_2\omega\int_0^\infty \bar{P}_n^{(2)}(x,s)\,\mu(x)dx\ \ n\ge 0 \quad (25)$$

Now multiplying equation (17) by z^n and add (18) and using the generating functions we get

$$\frac{\partial}{\partial x}\bar{P}_q(x,z,s)+\big(s+\lambda r_1(1-C(z))+\mu(x)\big)\bar{P}_q(x,z,s)=0 \quad (26)$$

$$\frac{\partial}{\partial x}\bar{V}_q(x,z,s)+\big(s+\lambda r_2(1-C(z))+v(x)\big)\bar{V}_q(x,z,s)=0 \quad (27)$$

$$(s+\lambda r_1(1-C(z))+\beta)\bar{R}_q^{(i)}(z,s)=\alpha z\left\{\int_0^\infty P_q^{(1)}(x,z,s)dx+\int_0^\infty P_q^{(1)}(x,z,s)dx\right\} \quad (28)$$

Similarly,

$$P_q^{(2)}(x,z,s)=\int_0^\infty P_q^{(1)}(x,z,s)\mu_1 dx \quad (28a)$$

Multiply bothsides of equation (24) by z^{n+1}sum over $n\ from\ 0\ to\infty$, and use the generating function defined above, to get

$$z\,r_2\bar{P}_q\ \ (0,z,s)=r_2(1-\omega)(pz+q)\int_0^\infty P_q^{(2)}(x,z,s)\mu_2(x)dx+r_1\int_0^\infty \bar{V}_q(x,z,s)v(x)dx +r_2\beta\bar{R}_q^{(i)}(z,s)+(1-s\bar{Q}(s))+\lambda r_1 r_2(C(s)-1)\bar{Q}(s) \quad (29)$$

Performing similar operation on equations (25) by $z^n$and sum over $0\ to\ \infty$ and use generating function defined above, we get

$$r_1\bar{V}_q(0,z,s)=r_2\omega\int_0^\infty \bar{P}_q^{(2)}(x,z,s)\mu_2(x)dx\ \ n\ge 0 \quad (30)$$

Integrating equation (37) between $0\ to\ x$,we get

$$\bar{P_q}^{(1)}(x,z,s)=\bar{P_q}^{(1)}(0,z,s)e^{-[s+\lambda r_1(1-C(z))+\alpha]x-\int_0^x \mu_1(t)dt} \quad (31)$$

$$\bar{P_q}^{(2)}(x,z,s)=\bar{P_q}^{(2)}(0,z,s)e^{-[s+\lambda r_1(1-C(z))+\alpha]x-\int_0^x \mu_1(t)dt} \quad (31a)$$

$where\bar{P}^{(1)}(0,z,s)$is given by equation (29)

Again integrating equation (31) by parts with respect to x yelds

$$\bar{P_q}^{(1)}(z,s)=\bar{P_q}^{(1)}(0,z,s)\left[\frac{1-\bar{B}_1(s+\lambda r_1(1-C(z))+\alpha)}{(s+\lambda r_1(1-C(z))+\alpha)}\right] \quad (32)$$

$$\bar{P_q}^{(2)}(z,s)=\bar{P_q}^{(2)}(0,z,s)\left[\frac{1-\bar{B}_2(s+\lambda r_1(1-C(z))+\alpha)}{(s+\lambda r_1(1-C(z))+\alpha)}\right] \quad (32a)$$

$$where\bar{B}_1\Big(s+\lambda\alpha\big(1-C(z)\big)\Big)=e^{-[s+\lambda\alpha(1-C(z))]x}dB_1(x) \quad (33)$$

$$\bar{B}_2\Big(s+\lambda\alpha\big(1-C(z)\big)\Big)=e^{-[s+\lambda\alpha(1-C(z))]x}dB_2(x) \quad (33a)$$

Is the Laplace-Stieltjes transform of the first stage of service $B(x)$. Now multiplying bothsides of equatons (31) by $\mu(x)$ over x we obtain

$$\int_0^\infty \bar{P}_q^{(1)}(x,z,s)\mu_1(x)dx = \bar{P}_q^{(1)}(0,z,s)\bar{B}_1\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right) \quad (34)$$

$$\int_0^\infty \bar{P}_q^{(2)}(x,z,s)\mu_1(x)dx = \bar{P}_q^{(2)}(0,z,s)\bar{B}_2\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right) \quad (34a)$$

Using Equation (34b)(28a),equation (30) becomes

$$r_1\bar{V}_q(0,z,s) = r_2\omega\bar{P}_q^{(2)}(0,z,s)\bar{B}_2\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)$$
$$= r_2\omega\bar{P}_q^{(2)}(0,z,s)\bar{B}_1\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)\bar{B}_2\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right) \quad (35)$$

Similarly,on integrating equations (27) from 0 $to\ x$,we get

$$\bar{V}_q^{(1)}(x,z,s) = \bar{V}_q^{(1)}(0,z,s)e^{-[s+\lambda r_2(1-C(z))+\alpha]x-\int_0^x v(t)dt} \quad (36)$$

Substituting the value of $\bar{V}_q(0,z,s)$ from equation(35) in equation (36) we get

$$\bar{V}_q(x,z,s) = \left(\frac{r_2}{r_1}\right)\omega\bar{P}_q^{(1)}(0,z,s)\bar{B}_1\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)$$
$$\bar{B}_2\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)e^{-[s+\lambda r_2(1-C(z))+\alpha]x-\int_0^x v(t)dt} \quad (37)$$

Again integrating equations (37) by parts with respect to x yields,

$$\bar{V}_q^{(1)}(z,s) = \left(\frac{r_2}{r_1}\right)\omega\bar{P}_q^{(1)}(0,z,s)\bar{B}_1\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)$$
$$\bar{B}_2\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)\left[\frac{1-\bar{V}\left(s+\lambda r_2\left(1-C(z)\right)\right)}{\left(s+\lambda r_2\left(1-C(z)\right)\right)}\right] \quad (38)$$

$$where\ \bar{V}\left(s+\lambda r_2\left(1-C(z)\right)\right) = e^{-[s+\lambda r_2(1-C(z))+\alpha]x}dV(x) \quad (39)$$

Is the Laplace-Stieltjes transform of the vacation time $V(x)$. Now multiplying bothsides of equation (37) by $V(x)$ and integrating over x, we get

$$\int_0^\infty \bar{V}_q(x,z,s)v(x)dx = \left(\frac{r_2}{r_1}\right)\omega\bar{P}_q^{(1)}(0,z,s)\bar{B}_1\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)$$
$$\bar{B}_2\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)\bar{V}\left(s+\lambda r_2\left(1-C(z)\right)\right) \quad (40)$$

Using equation (32)(32a),equation (28) reduces to

$$\bar{R}_q^{(i)}(z,s) = \frac{\begin{matrix}\alpha z\bar{P}_q^{(1)}(0,z,s)\bar{B}_1\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)\\ +\bar{B}_1\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)\left(1-\bar{B}_2\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)\right)\end{matrix}}{\left[\left(s+\lambda r_1\left(1-C(z)\right)+\alpha\right)\right]\left[s+\lambda r_1\left(1-C(z)\right)+\beta\right]} \quad (41)$$

Now using equations (34),(34a),(40) and (41) in equation (29) and solving for$\bar{P}_q^{(1)}(0,z,s)$, we get

$$zr_2\bar{P}_q^{(1)}(0,z,s) = r_2(1-\omega)(pz+q)\bar{P}_q^{(1)}(0,z,s)\bar{B}_1\left(f_1(z)\right)\bar{B}_2\left(f_1(z)\right)$$
$$+r_2\omega\bar{P}_q^{(1)}(0,z,s)\bar{B}_1\left(f_1(z)\right)\bar{B}_2\left(f_1(z)\right)\bar{V}\left(s+\lambda r_2\left(1-C(z)\right)\right)$$
$$+\frac{r_2\beta\alpha z\bar{P}_q^{(1)}(0,z,s)\bar{B}_1\left(f_1(z)\right)\bar{B}_2\left(f_1(z)\right)}{f_1(z)f_2(z)} + \lambda r_1 r_2(C(z)-1)\bar{Q}(s) + \left(1+S\bar{Q}(s)\right)$$

$$\bar{P}_q^{(1)}(0,z,s)\begin{bmatrix} f_1(z)(z)f_2(z)(1-\omega)(pz+q)\bar{B}_1\left(f_1(z)\right)\bar{B}_2\left(f_1(z)\right) \\ -\omega\bar{B}_1\left(f_1(z)\right)\bar{B}_2\left(f_1(z)\right) - \beta\alpha z\left(1-\bar{B}_1\left(f_1(z)\right)\bar{B}_2\left(f_1(z)\right)\right)\end{bmatrix}$$

$$= \frac{f_1(z)f_2(z)\left[\lambda r_1(C(z)-1)\bar{Q}(s) + \frac{1}{r_2}\left(1-S\bar{Q}(s)\right)\right]}{Dr} \quad (42)$$

$$Dr = \left(f_1(z)f_2(z)\right)\begin{bmatrix} z-(1-\omega)(pz+q)\bar{B}_1\left(f_1(z)\right)\bar{B}_2\left(f_1(z)\right) \\ -\omega\bar{B}_1\left(f_1(z)\right)\bar{B}_2\left(f_1(z)\right)\bar{V}\left(s+\lambda r_2\left(1-C(z)\right)\right) \\ -\beta\alpha z\left(1-\bar{B}_1\left(f_1(z)\right)\bar{B}_2\left(f_1(z)\right)\right)\end{bmatrix}$$

$$f_1(z) = s+\lambda r_1\left(1-C(z)\right)+\alpha\ ;\ f_2(z) = s+\lambda r_1\left(1-C(z)\right)+\beta;$$

Substituting the value of $\bar{P}_q^{(1)}(0,z,s)$ from equation(42) into equations (32),(38) and (41) we get

$$\bar{P}_q^{(1)}(z,s) = \frac{f_2(z)\left[\lambda r_1(C(z)-1)\left[1-\bar{B}_1(f_1(z))\bar{Q}(s)+\frac{1}{r_2}(1-S\bar{Q}(s))\right]\right]}{DR} \quad (43)$$

$$\bar{P}_q^{(2)}(z,s) = \frac{f_2(z)\left[\lambda r_1(C(z)-1)\left[1-\bar{B}_1(f_1(z))\bar{Q}(s)+\frac{1}{r_2}(1-S\bar{Q}(s))\right]\right]\left[\bar{B}_1(f_1(z))-\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\right]}{DR} \quad (43a)$$

$$\bar{V}_q(z,s) = \frac{\left\{\left(\frac{r_2}{r_1}\right)\omega f_1(z)f_2(z)\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\begin{bmatrix}\frac{1}{r_2}(1-S\bar{Q}(s))\\+\lambda r_1(C(z)-1)\bar{Q}(s)\end{bmatrix}\left[\frac{1-\bar{V}(s+\lambda r_2(1-C(z)))}{s+\lambda r_2(1-C(z))}\right]\right\}}{Dr} \quad (44)$$

$$\bar{R}_q(z,s) = \frac{\alpha z\left[\lambda r_1(C(z)-1)\bar{Q}(s)+\frac{1}{r_2}(1-S\bar{Q}(s))\left[1-\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\right]\right]}{Dr} \quad (45)$$

V. THE STEADY STATE RESULTS

In this section, we shall derive the steady state prbability distribution for our queueing model.This can be obtained by applying the well-known property.

$$\lim_{s\to 0} s\bar{f}(s) = \lim_{t\to\infty} f(t) \quad (46)$$

Multiplying both sides of equations (43)(43a),(44),(45) by s,taking limit as s→0,applying property (46) and simplifying,we obtain

$$P_q^{(1)}(z) = \frac{f_2(z)\left[1-\bar{B}_1(f_1(z))\right][\lambda r_1(C(z)-1)Q]}{Dr} \quad (47)$$

$$P_q^{(2)}(z) = \frac{f_2(z)\left[\bar{B}_1(f_1(z))-\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\right][\lambda r_1(C(z)-1)Q]}{Dr} \quad (48)$$

$$V_q(z) = \frac{\omega f_1(z)f_2(z)\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\left[\bar{V}\left(\lambda r_2(1-C(z))\right)-1\right]Q}{Dr} \quad (49)$$

$$R_q^{(i)}(z) = \frac{\alpha z\left[1-\bar{B}_2(f_1(z))\bar{B}_1(f_1(z))\right][[\lambda r_1(C(z)-1)Q]]}{Dr} \quad (50)$$

$$S_q(z) = P_q^{(1)}(z) + P_q^{(2)}(z) + V_q(z) + R_q^{(i)}(z)$$

$$= \frac{\lambda r_1(C(z)-1)Q\begin{bmatrix}f_2(z)-f_2(z)\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\\+\alpha z-\alpha z\bar{B}_2(f_1(z))\bar{B}_1(f_1(z))\\+\omega f_1(z)f_2(z)\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))Q\left[\bar{V}\left(\lambda r_2(1-C(z))\right)-1\right]\end{bmatrix}}{Dr} \quad (51)$$

In order to obtain Q, using the normalization condition, as follows

$$S_q(1) + Q = 1 \quad (52)$$

We see that $z = 1, S_q(z)$ is indeterminate of the form $0/0$. We apply L'Hospital rule in equation (51)

$$S_q(1) = \frac{\begin{matrix}\lambda r_1 E(I)[\beta - \beta \bar{B}_1(\alpha)\bar{B}_2(\alpha) + \alpha - \alpha \bar{B}_2(\alpha)\bar{B}_1(\alpha)] \\ +\omega Q E(I)E(v)\bar{B}_1(\alpha)\bar{B}_2(\alpha)\lambda r_2\end{matrix}}{\begin{matrix}\lambda r_1 E(I)\left(1 - \bar{B}_1(\alpha)\bar{B}_2(\alpha)\right)(\alpha+\beta) \\ +\alpha\beta \left\{\begin{matrix}1 - \left[(1-\omega)\left(p\bar{B}_1(\alpha)\bar{B}_2(\alpha)\lambda r_1 E(I)\right)\right][\overline{B'}_1(\alpha)\bar{B}_2(\alpha) + \bar{B}_1(\alpha)\overline{B'}_2(\alpha)] \\ +\omega\lambda r_1 r_2 E(I)E(v)\bar{B}_1(\alpha)\bar{B}_2(\alpha)\end{matrix}\right\} \\ -\beta\alpha - \beta\alpha\bar{B}_1(\alpha)\bar{B}_2(\alpha) + \beta\alpha(\lambda r_1 E(I))\left(\left[\overline{B'}_2(\alpha)\bar{B}_1(\alpha) + \bar{B}'_1(\alpha)\bar{B}_2(\alpha)\right]\right)\end{matrix}} \quad (53)$$

$$where\ Dr = \left\{ f_1(z)f_2(z)\begin{bmatrix} z - (1-\omega)(pz+q)\bar{B}_1(\alpha)\bar{B}_2(\alpha) - \omega\bar{B}_1(\alpha)\bar{B}_2(\alpha) \\ \bar{V}\left(\lambda r_2\left(1 - C(z)\right)\right)\end{bmatrix}\right\} \\ -\beta\alpha z\left(1 - \bar{B}_2(\alpha)\bar{B}_1(\alpha)\left(f_1(z)\right)\right)$$

And $\bar{B}(0) = 1, -C'(1) = E(I), V'(0) = E(V)\ \ C(1) = 1$

Using equation (53) in equation (52)

$$Q = 1 - \frac{\begin{matrix}\lambda r_1 E(I)[\beta - \beta \bar{B}_1(\alpha)\bar{B}_2(\alpha) + \alpha - \alpha \bar{B}_2(\alpha)\bar{B}_1(\alpha)] \\ +\omega Q E(I)E(v)\bar{B}_1(\alpha)\bar{B}_2(\alpha)\lambda r_2\end{matrix}}{\begin{matrix}\lambda r_1 E(I)\left(1 - \bar{B}_1(\alpha)\bar{B}_2(\alpha)\right)(\alpha+\beta) \\ +\alpha\beta \left\{\begin{matrix}1 - \left[(1-\omega)\left(p\bar{B}_1(\alpha)\bar{B}_2(\alpha)\lambda r_1 E(I)\right)\right][\overline{B'}_1(\alpha)\bar{B}_2(\alpha) + \bar{B}_1(\alpha)\overline{B'}_2(\alpha)] \\ +\omega\lambda r_1 r_2 E(I)E(v)\bar{B}_1(\alpha)\bar{B}_2(\alpha)\end{matrix}\right\} \\ -\beta\alpha - \beta\alpha\bar{B}_1(\alpha)\bar{B}_2(\alpha) + \beta\alpha(\lambda r_1 E(I))\left(\left[\overline{B'}_2(\alpha)\bar{B}_1(\alpha) + \bar{B}'_1(\alpha)\bar{B}_2(\alpha)\right]\right)\end{matrix}} \quad (54)$$

And the Utilization factor ρ of the system is given by

$$\rho = \frac{\begin{matrix}\lambda r_1 E(I)[\beta - \beta \bar{B}_1(\alpha)\bar{B}_2(\alpha) + \alpha - \alpha \bar{B}_2(\alpha)\bar{B}_1(\alpha)] \\ +\omega Q E(I)E(v)\bar{B}_1(\alpha)\bar{B}_2(\alpha)\lambda r_2\end{matrix}}{\begin{matrix}\lambda r_1 E(I)\left(1 - \bar{B}_1(\alpha)\bar{B}_2(\alpha)\right)(\alpha+\beta) \\ +\alpha\beta \left\{\begin{matrix}1 - \left[(1-\omega)\left(p\bar{B}_1(\alpha)\bar{B}_2(\alpha)\lambda r_1 E(I)\right)\right][\overline{B'}_1(\alpha)\bar{B}_2(\alpha) + \bar{B}_1(\alpha)\overline{B'}_2(\alpha)] \\ +\omega\lambda r_1 r_2 E(I)E(v)\bar{B}_1(\alpha)\bar{B}_2(\alpha)\end{matrix}\right\} \\ -\beta\alpha - \beta\alpha\bar{B}_1(\alpha)\bar{B}_2(\alpha) + \beta\alpha(\lambda r_1 E(I))\left(\left[\overline{B'}_2(\alpha)\bar{B}_1(\alpha) + \bar{B}'_1(\alpha)\bar{B}_2(\alpha)\right]\right)\end{matrix}} \quad (55)$$

Where $\rho < 1$ is the stability condition under which the steady state exists.

VI. THE MEAN QUEUE SIZE AND THE MEAN SYSTEM SIZE

Let L_q denote the mean number of customers in the queue under the steady state. Then

$$L_q = \lim_{z\to 1}\frac{d}{dz}S_q(z) = \lim_{z\to 1}\left[\frac{D'(1)N''(1) - N'(1)D''(1)}{2\left(D'(1)\right)^2}\right] \quad (56)$$

$$N'(1) = \lambda r_1 E(I)[\beta - \beta\bar{B}_1(\alpha)\bar{B}_2(\alpha) + \alpha - \alpha\bar{B}_2(\alpha)\bar{B}_1(\alpha)] \\ +\omega Q E(I)E(v)\bar{B}_1(\alpha)\bar{B}_2(\alpha)\lambda r_2 \quad (57)$$

$$N''(1) = \lambda r_1 Q[E(I(I-1))[\beta - \beta\bar{B}_1(\alpha)\bar{B}_2(\alpha) + \alpha - \alpha\bar{B}_1(\alpha)\bar{B}_2(\alpha)] \\ -E(I)[\lambda r_1 E(I) - \lambda r_1 E(I)\bar{B}_1(\alpha)\bar{B}_2(\alpha) - \beta\overline{B'}_1(\alpha)\lambda r_1 E(I)\bar{B}_2(\alpha) \\ -\beta\bar{B}_1(\alpha)\overline{B'}_2(\alpha)\lambda r_1 E(I) + \alpha - \alpha\bar{B}_2(\alpha)\bar{B}_1(\alpha) - (\alpha+\beta)\lambda r_1 E(I)] \\ +\omega Q[\lambda^2 r_1 r_2 E(I)^2\overline{B'}_1(\alpha)\beta\bar{B}_2(\alpha)E(V) + \lambda^2 r_1 r_2 E(I)^2\overline{B'}_1(\alpha)\beta\bar{B}_2(\alpha)E(V) \\ +E(I)^2\lambda^2 r_1 r_2\alpha\beta\bar{B}_1(\alpha)\bar{B}_2(\alpha) + \alpha\beta\lambda^2 r_1 r_2\overline{B'}_2(\alpha)\bar{B}_1(\alpha)E(V) \\ +\lambda r_2 E\left(I(I-1)\right)E(V)\alpha\beta\bar{B}_1(\alpha)\bar{B}_2(\alpha) - \lambda^2 E(I)^2\beta\bar{B}_1(\alpha)\bar{B}_2(\alpha)E(V)r_1 \\ -\lambda^2 r_1 r_2 E(I)^2\alpha\bar{B}_1(\alpha)\bar{B}_2(\alpha)E(V) - \alpha\beta\overline{B'}_1(\alpha)\lambda^2 E(I)^2\bar{B}_2(\alpha)E(V)r_1 r_2 \\ -\alpha\beta\overline{B'}_2(\alpha)\lambda^2 E(I)^2\bar{B}_1(\alpha)E(V)r_1 r_2 - \lambda^2 r^2 E(I)^2\alpha\bar{B}_1(\alpha)\bar{B}_2(\alpha)E(V^2)] \quad (58)$$

$$D'(1) = \lambda r_1 E(I)\left(1 - \bar{B}_1(\alpha)\bar{B}_2(\alpha)\right)(\alpha+\beta)$$
$$+\alpha\beta\left\{\begin{matrix} 1 - \left[(1-\theta)\left(p\bar{B}_1(\alpha)\bar{B}_2(\alpha)\lambda r_1 E(I)\right)\right]\left[\overline{B'}_1(\alpha)\bar{B}_2(\alpha) + \bar{B}_1(\alpha)\overline{B'}_2(\alpha)\right] \\ +\theta\lambda r_1 r_2 E(I)E(v)\bar{B}_1(\alpha)\bar{B}_2(\alpha) \end{matrix}\right\} \quad (59)$$
$$-\beta\alpha - \beta\alpha\bar{B}_1(\alpha)\bar{B}_2(\alpha) + \beta\alpha(\lambda r_1 E(I))\left(\left[\overline{B'}_2(\alpha)\bar{B}_1(\alpha) + \bar{B}'_1(\alpha)\bar{B}_2(\alpha)\right]\right)$$

$$D''(1) = -\lambda r_1 E\left(I(I-1)\right)(\beta+\alpha)[1 - \bar{B}_1(\alpha)\bar{B}_2(\alpha)]$$
$$+2\lambda^2 r_1{}^2[E(I)]^2[1 - \bar{B}_1(\alpha)\bar{B}_2(\alpha)] + \lambda r_1 E(I)(\beta+\alpha)$$
$$[1 - (1-\omega)p\bar{B}_1(\alpha)\bar{B}_2(\alpha) - \omega\lambda r_2 E(I)E(V)\bar{B}_1(\alpha)\bar{B}_2(\alpha)]$$
$$+\lambda r_1(\beta+\alpha)E(I)\left[1 - (1-\omega)(p\bar{B}_1(\alpha)\bar{B}_2(\alpha) + \lambda r_1 E(I)\begin{bmatrix}\overline{B'}_1(\alpha)\bar{B}_2(\alpha) \\ +\bar{B}_1(\alpha)\overline{B'}_2(\alpha)\end{bmatrix}\right]$$
$$+\alpha\beta[-(1-\omega)\{p\lambda r_1 E(I)[\overline{B'}_1(\alpha)\bar{B}_2(\alpha) + \bar{B}_1(\alpha)\overline{B'}_2(\alpha)]$$
$$+p\lambda r_1 E(I)[\overline{B'}_1(\alpha)\bar{B}_2(\alpha) + \bar{B}_1(\alpha)\overline{B'}_2(\alpha)]$$
$$+\left(-\lambda r_1 E\left(I(I-1)\right)\right)[\overline{B'}_1(\alpha)\bar{B}_2(\alpha) + \bar{B}_1(\alpha)\overline{B'}_2(\alpha)]$$
$$+\lambda r_1 E(I)\left[\begin{matrix}\overline{B''}_1(\alpha)\bar{B}_2(\alpha) + 2\overline{B'}_1(\alpha)\overline{B'}_2(\alpha) \\ \bar{B}_1(\alpha)\overline{B''}_2(\alpha)\end{matrix}\right]\}$$
$$+\omega r_1 r_2 \lambda\left(E(I)\right)^2 E(V)[\overline{B'}_1(\alpha)\bar{B}_2(\alpha) + \bar{B}_1(\alpha)\overline{B'}_2(\alpha)]$$
$$-\omega\lambda r_2 E\left(I(I-1)\right)E(V)\bar{B}_1(\alpha)\bar{B}_2(\alpha) - \lambda^2 r^2\left(E(I)\right)^2 E(V^2)\bar{B}_1(\alpha)\bar{B}_2(\alpha)$$
$$+\omega\lambda^2 r_1 r_2\left(E(I)\right)^2 E(V)[\overline{B'}_1(\alpha)\bar{B}_2(\alpha) + \bar{B}_1(\alpha)\overline{B'}_2(\alpha)] \quad (60)$$

Where primes and double primes in (56) denote the first and second derivatives at $z = 1$, respectively. Carrying out the derivatives at $z = 1$ and if we substitute the values of $N'(1), N''(1), D'(1)\ and D''(1)$ into (56), we obtain L_q in closed form. Further, the mean waiting time of a customer could be found using $W_q = L_q/\lambda$ and other performance measures can be determined using Little's formula.

If we substitute the values $N'(1), N''(1), D'(1), D''(1)$ into equations (56) we obtain L_q in the closed form.

Further, we find the mean system size L using Little's formula. Thus we have

$$L = L_q + \rho \quad (61)$$

Where Lq has been found in equation (56).

VII. CONCLUSIONS

In this paper we have studied a batch arrival of restricted admissibility, two types of service, feedback service with Bernoulli vacation and optional repair. This paper clearly analyses the transient solution, steady state results, and some performance measures of the queueing system. Further, this model can be extended by incorporating the concept Delay time, extended vacation and reneging.

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